ADDITIONAL MATHEMATICS

Paper 4037/11
Paper 11

Key Messages

In order to succeed in this paper, candidates need to have a full understanding of all the topics on the syllabus. They need to read the questions carefully in order to ensure that they are answering the question asked, using the required method and giving their answer in the form required, where these are specified.

General Comments

The examination provided the candidates with plenty of opportunity to display their skills. There was no evidence that the examination was of an inappropriate length. A few candidates omitted questions or parts of questions but this appeared to be a consequence of a lack of knowledge rather than any timing issue. Candidates should be advised not to do their working in pencil and then overwrite it in pen, as this can make it difficult for Examiners to read resulting in a possible loss of credit.

Comments on Specific Questions

Question 1

The majority of candidates were able to gain full marks on this question. Most worked from the left hand side of the identity to obtain a single fraction. Those who were unable to complete the process usually omitted to cancel $(1+\sin\theta)$ to get the final answer.

Question 2

(i) As the question stated "show", it was necessary for candidates to evaluate the moduli of **a** and **b** + **c**. It was not sufficient to state that $\begin{vmatrix}4\\3\end{vmatrix}=\begin{vmatrix}-3\\4\end{vmatrix}$.

(ii) Most candidates gained full marks on this question, and only a few were unable to obtain and solve the two linear equations.

Answer: –49, 80.5

Question 3

(a) Almost all candidates were able to answer part **(i)** correctly. Parts **(ii)** and **(iii)** caused more problems, with a number of candidates also shading the region outside $A \cup B \cup C$.

(b) A number of candidates did not appear to be aware of the meaning of the notation n( ).

(i) The answer "0 or –2" was not acceptable.

(ii) $\varnothing$ or "the empty set" were not acceptable answers.

Answer: **(i)** 2 **(ii)** 0.

Question 4

The majority of candidates were able to gain marks on this question, and virtually all realised the need to eliminate y and use the discriminant of the resulting quadratic equation in x. There were a few attempts to equate the gradients of the line and the curve but these made little progress. Some candidates who managed to obtain the critical values of 3 and 4 were unable to identify the correct range of values of k.

Answer: $3 < k < 4$

Question 5

(i) Only a minority of candidates were able to obtain the fully correct answer to this question, and many attempted solutions included e^{2x} rather than e^{x^2}.

(ii) For those who got the correct answer to part **(i)** this was an easy 2 marks, otherwise it was difficult to make progress.

(iii) Candidates were required to show that they were using limits correctly. A correct answer from a calculator following wrong working was not allowed.

Answer: **(i)** $2xe^{x^2}$ **(ii)** $\frac{1}{2}e^{x^2}$ **(iii)** 26.8

Question 6

(i) The majority of candidates were able to obtain the correct 3 x 2 matrix.

(ii) The method for finding the inverse of a 2 x 2 matrix was well known.

(iii) Candidates were asked to use their answer to part **(ii)** to solve these equations, and solution of the equations by methods other than the matrix method required were not awarded full marks. A number of candidates post-multiplied, rather than pre-multiplied $\begin{pmatrix} -3 \\ -22 \end{pmatrix}$ by $\mathbf{B}^{-1}$, usually resulting in an incorrect answer.

Answer: **(i)** $\begin{pmatrix} 10 & 19 \\ 32 & 37 \\ 14 & 14 \end{pmatrix}$ **(ii)** $\frac{1}{7}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$ **(iii)** $x = 0.5$, $y = -2.5$

Question 7

(i) Many candidates were able to gain full marks on this question, although a few omitted consideration of the constant of integration. A common incorrect approach was to find the equation of a straight line through the point $\left(\frac{1}{2}, \frac{5}{6}\right)$ rather than a curve.

(ii) Again, many candidates were able to gain full marks here, although a number of them chose to use the coordinates $\left(\frac{1}{2}, \frac{5}{6}\right)$ rather than (1, 2.5).

Answer: **(i)** $y = 2x^2 - \frac{1}{x+1} + 1$ **(ii)** $34y + 8x = 93$

Question 8

(i) There were many fully correct graphs, although many of these utilised lg rather than ln. However, there were also many candidates who were unable to produce an appropriate straight line graph. A small number of candidates produced correct graphs with axes transposed, which led to problems in subsequent parts of the question. Use of difficult scales, particularly on the lg p-axis, made accurate plotting difficult.

(ii) Candidates should be aware that, when calculating the gradient of the line, values from the table should only be used when those points lie on the plotted graph.

(iii) Use of the graph to find the value of lg p corresponding to lg 35 was the easiest and most successful way of answering this question. When other methods were used, lg k was often incorrectly deduced from the intercept by candidates who only drew a horizontal axis starting at either 0.5 or 1. Many candidates were also confused as to whether the intercept was k or lg k.

Answer: **(ii)** –1.5 **(iii)** 14

Question 9

(a) Many candidates knew to evaluate the area under the graph to get the distance travelled. A common mistake was to take the reading on the v-axis to be either 14 or 10.4 rather than 12.

(b) The majority of candidates plotted a velocity of 0 between $t = 6$ and $t = 25$, but it was common to see straight lines joining (0, 0) to (6, 2) and (25, 0) to (30, 1).

(c) (i) This question was generally well done, and most candidates attempted to differentiate to find an expression for v, and to equate this to 0.

(ii) As in part **(i)**, this was done well, although there were a few inappropriate attempts to use the constant acceleration formulae.

Answer: **(a)** 480 **(c)** **(i)** 3 **(ii)** 7

Question 10

(a) Candidates found this to be a difficult question and there were few completely correct answers. Many misinterpreted the question and looked only for 3 digit numbers.

(b) The majority of candidates were able to gain full marks on this question.

(c) Only a few candidates were able to gain full marks on this question. Credit was given to those who realised that the number of committees with the oldest man only was equal to the number with the oldest woman only.

Answer: **(a)** 28 **(b) (i)** 420 **(ii)** 240

Question 11

(a) Most candidates were able to reduce the equation to $\tan 2x = -0.6$, although there were a number of attempts, using incorrect identities, to reduce the equation to $\sin x =$ or $\cos x =$. Candidates should be aware that the correct procedure to solve an equation of this type is to find values of $2x$ first, before dividing by 2. Those using a calculator to obtain –31° leading to –15.5° were unable to obtain all the valid solutions.

(b) This question was generally well done, although a number of candidates chose to use incorrect substitutions such as $\operatorname{cosec}^2 y + 1 = \cot^2 y$ or $\operatorname{cosec} y = \dfrac{1}{\cos y}$.

(c) As in part **(a)**, many candidates were able to get as far as $z + 1.2 = 0.841$. Again, the correct procedure is to obtain values of $z + 1.2$ before subtracting 1.2. Candidates who worked with $z = -0.36$ rather than 0.841 were invariably unable to get both correct answers.

Answer: **(a)** 74.5°, 164.5° **(b)** 210°, 330° **(c)** 4.24, 5.92

ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key Messages

In order to succeed in this paper, candidates need to have a full understanding of all the topics on the syllabus. They need to read the questions carefully in order to ensure that they are answering the question asked, using the required method and giving their answer in the form required, where these are specified.

General Comments

The examination provided the candidates with plenty of opportunity to display their skills. There was no evidence that the examination was of an inappropriate length. A few candidates omitted questions or parts of questions but this appeared to be a consequence of a lack of knowledge rather than any timing issue. Candidates should be advised not to do their working in pencil and then overwrite it in pen, as this can make it difficult for Examiners to read resulting in a possible loss of credit.

Candidates continue to lose marks due to prematurely rounding results. Clarity of numbers and lettering would also have helped as sometimes it was difficult to be certain of the intentions of some candidates; making r and π look very similar was a prime example. There were also marks lost due to carelessness with signs on terms, especially when expanding brackets and also with manipulation of terms when solving an equation. It is unfortunate to have completed a correct integration or solved a trigonometric equation only to lose marks in the final line of the solution by writing $3a = \frac{\pi}{3}$ so $a = \pi$, or $a = \frac{\pi}{6}$, an all too common error in **Question 4**, or $2z = \frac{\pi}{2}$, so $z = \pi$, an all too common error in **Question 11(c)**.

Comments on Specific Questions

Question 1

This question was mostly well done, with nearly all candidates able to gain marks. The majority used the expected approach of forming a single fraction, expanding out and simplifying the numerator. They then made use of the appropriate trigonometric identity and used factorisation to obtain the required result. As a result, many were completely successful. However, some candidates using this approach missed out a step by omitting to show the factorisation, obviously being able to do it in their heads. The question does ask candidates to 'Show' so it is important that candidates are encouraged to show each step of their working. On this occasion there were very few alternative approaches, but those who used other methods generally did well.

Question 2

(a) It was expected that in the first diagram two sets would be drawn that did not meet; usually this was seen, although a few candidates did make them (incorrectly) touch. In the second diagram the two sets needed to be drawn with one set completely inside the other. Most candidates did this but some then went on to incorrectly label their sets. Candidates need to become familiar with the less used set notation on the syllabus.

(b) Most candidates realised that this part of the question could be solved by the drawing of a simple Venn diagram, filling in the information given and then deducing the information required. Many candidates scored full marks, with the vast majority getting part **(ii)** correct. In part **(i)** the most common error was for the answer to be given as 2, the number of elements in set *Q* only. In part **(iii)** incorrect answers of 13 (the number of elements in set *P*) and 14 (the number of elements in the complement of set *Q*) were common examples of errors made.

Solution: **(b)(i)** 6 **(ii)** 5 **(iii)** 9

Question 3

(i) Many candidates were clearly familiar with the drawing of the graphs of modulus functions. The standard of the sketches produced was variable, with many candidates not appreciating the symmetrical aspect of the centre part of the curve. In spite of the request to do so, many candidates did not label all the points of intersection of the curve with the coordinate axes. The given domain also meant that the outer two parts of the curve extended well above the maximum point on the curve, an aspect that many candidates did not consider.

(ii) There was variable understanding of this question, with many candidates simply finding the values of x where $y = 0$. Candidates who did understand the question were most successful if they had already found the vertex by symmetry when drawing the graph in part **(i)**. Other approaches included the use of the discriminant of the quadratic equation involved, not an easy route, with varying degrees of success. As a result, there were many algebraic and numerical errors in applying the formula for the discriminant, together with a lot of confusion in how to present the required answer.

Solution: **(ii)** $k > \frac{25}{8}$

Question 4

Almost all candidates attempted an integration with the vast majority obtaining the correct result of $-\frac{2}{3}\cos 3x$. Common errors included $-2\cos 3x$, $-6\cos 3x$ and even $6\cos 3x$. Very few candidates worked only with the original sine function, making no attempt at integration at all, but there were several candidates who made no response in any form. Apart from the usual arithmetic slips, the main errors came from either applying incorrect limits or from ignoring / misinterpreting the zero lower limit. The incorrect limits seen were frequently 0 to 1 or sometimes 0 to $\frac{\pi}{3}$. A surprising number of candidates successfully arrived at $3a = \frac{\pi}{3}$ and then offered solutions of $a = \pi$ or $a = \frac{\pi}{6}$. Too many candidates displayed imprecise or careless notation, for example, the integral sign appearing in lines of working after integration had been performed. Not presenting answers to 3 significant figures, as required – in this case usually 0.35 instead of 0.349 – was a common feature in responses throughout the paper. A few candidates offered the alternative legitimate response that involved integrating from a to $\frac{\pi}{3}$ and some of the more successful scripts even included both interpretations. Overall, this question was reasonably well done considering the possible opportunities for error and confusion. Those who made an initial sketch tended to obtain the correct outcome. There were many completely correct responses.

Solution: $\frac{\pi}{9}$ or 0.349

Question 5

(i) Most candidates began correctly by putting the expressions to the same base, writing 4^y as 2^{2y} and $\frac{1}{8}$ as 2^{-3}. Not every candidate then combined the two terms to obtain 2^{5x+2y} before showing the given final result. Sometimes the addition sign appeared prematurely which gave doubt as to whether the candidate fully understood the principle being tested. A few tried to apply logarithms with mixed success. Merely writing $\log 4^y$ and then changing that to $2y$ did not demonstrate the knowledge required, especially when the answer was given in the question.

(ii) There were very many completely correct solutions to this part. Usually 49^{2y} was dealt with correctly but on quite a few occasions candidates treated 1 as 7^1 and produced an incorrect equation of $x + 4y = 1$ which actually made the subsequent work slightly more difficult. A few candidates tried to follow the 'principle' from part **(i)** for y and changed 49^{2y} into $7^{7\times 2y}$ which was not a success. Almost every candidate was able to correctly solve their two equations, although those not using $x + 4y = 0$ lost accuracy.

Solution: **(ii)** $x = -\frac{2}{3}$, $y = \frac{1}{6}$

Question 6

(a) The order for multiplication seemed irrelevant to many candidates. It was not unusual to see either **YX** or **ZY,** along with the same matrices written down in full but the wrong way round. It is necessary that candidates recognise the importance of order when considering matrix multiplication. Many candidates wasted time and effort by evaluating the matrix products in spite of a request not to do so.

(b) Out of those candidates who managed to earn full marks for this question, the majority used the inverse matrix correctly in order to find the unknown matrix **B**. There were fewer instances of post multiplication than pre-multiplication than in the past. Most candidates were able to perform basic matrix multiplication correctly, with the occasional slip. Some candidates chose to use algebraic methods, forming 4 equations with 4 unknowns going on to solve them, usually correctly.

Solution: **(a) YX**, **ZY** **(b)** $\begin{pmatrix} 3 & -1 \\ 6 & -7 \end{pmatrix}$

Question 7

(i) Most candidates made use of either the cosine rule or the sine of the half angle $\frac{\theta}{2}$. A small number of candidates chose to equate the area of the triangle in two different forms. The majority of candidates were successful in finding an appropriate method but few appreciated that working with figures of an accuracy of at least 4 decimal places, so that the resulting answer could then be rounded to 3 decimal places was necessary. Converting between degrees and radians tended to cause unnecessary confusion.

(ii) It was essential that candidates had a clear plan of what was required. Common errors resulting from the omission to do this involved adding extra, unnecessary, lengths or using an incorrect angle, for example, 1.696 radians rather than $2\pi - 1.696$ radians. Candidates who used degrees tended to mix them with π and/or radians in incorrect expressions. There was some premature rounding but most candidates obtained a correct major arc length with the majority going on to produce a completely correct solution.

(iii) There were many fully correct responses but there were also candidates who formed a poor plan that did not relate to the diagram. The area of the sector was more successfully found than the area of the triangle, which tended to be forgotten rather than miscalculated.

Solution: **(ii)** 48.7 **(iii)** 179

Question 8

More practice in tackling problems of this type, using examples that do not merely involve the simple application of formulae, could be beneficially applied in classrooms. Mixing ^nP_r and ^nC_r situations could assist in identifying what is required in analysing similar problems.

(a)(i) Most candidates correctly found 720, either by working from first principles or by using $^6P_5$, though many did not show their method. The most common incorrect solution was from using $^6C_5$.

(i) Many candidates appreciated that as there were two even digits from the six, the solution should be a third of the answer to part **(i)** and showed that this was what they were attempting. Sadly some decided that as a half of all numbers are even they would divide 720 by two. Many went back to basics with plenty of success and it was not uncommon to see $^5P_4$ x 2. However, too many candidates showed a basic misunderstanding by stating how many of the digits were even rather than how many even numbers were generated by their combinations.

(ii) Those candidates who worked from basics often got a complete or part solution. The main stumbling block seemed to be that once the digit to provide the even number status was decided and the first digit had been selected, many still tried to choose from 5 remaining digits. Selecting 3 from the remaining 4 was the key and the Examiners credited use of $^4C_3$ or its equivalent even if not a complete solution. Many did reach 144 as their final answer, but often it became part of some confused calculations casting doubt on the level of understanding. An alternative approach which also yielded success was to start with 240 and subtract the total quantity of even numbers that were greater than 6000, i.e. 2 x 48.

(b) The majority of candidates found this part of the question difficult to deal with. Either very large numbers were obtained by treating this as a permutation problem rather than a combination one, or the twins were considered to be a single entity and $^{17}C_5$ or $17C_6$ entered the calculations. The numbers 1820, 8008 and 18564 were seen often enough to believe that the question was accessible to most candidates. In spite of the relative difficulty, a pleasing number of candidates did obtain the final correct answer of 9828

Solution: **(a)(i)** 720 **(ii)** 240 **(iii)** 144 **(b)** 9828

Question 9

(i) This part of the question was well answered by the majority of the candidates. The only real errors occurred when candidates appeared not to know the formula for the surface area of a cylinder. The height of the cylinder in terms of the radius was usually found using the given volume. Subsequent substitutions were then usually correctly applied. Candidates need to be advised to present their work clearly since many appeared to cross out all their terms in their attempts with cancellation and with the answer given this could lose marks.

(ii) This part of the question was also well done with many fully correct responses. The derivative of the given result from part **(i)** was usually correct and most candidates were then able to find a value for the radius when the derivative was equated to zero. Most evaluated this to 8.60 but it was often seen left in an unsimplified root form. Occasionally, candidates did not attempt to find the value of the area for their radius, another case of candidates making sure that they are answering the demands of the question. There also appeared many cases where candidates went straight to the second derivative showing the value to be a minimum without actually evaluating the area or finding the actual value of the radius. These candidates were not penalised for what was clearly a valid method.

Solution: **(ii)** 1395 or 1390

Question 10

There were a large number of marks available for this question on vectors, but unfortunately many candidates were able to gain either none or very few of them. More practice with work on vectors would be beneficial to most candidates.

(i) A number of correct alternative methods were used to find the velocity of the ship. The most common of these was for the candidate to find the length of the direction vector, then divide the speed by this length, before multiplying by the direction vector. Some candidates, however, simply multiplied the direction vector by 2 without showing where the multiple of 2 had come from. As the answer was given in the question this was not sufficient working to gain the marks. Those candidates that took the approach of showing that the given velocity has a magnitude of 26 often gained only one of the available marks because they did not go on to show that the given vector was in the direction stated.

(ii) Many candidates correctly multiplied the velocity vector by 4, although far too many of them then went on to incorrectly add the direction vector to this result, thus showing a basic misunderstanding of the situation.

(iii) Many candidates correctly added *t* times the velocity vector to their answer to part **(ii)**, although some candidates used the direction vector rather than the velocity vector and others simply multiplied their answer to part **(ii)** by *t*. Again, a basic misunderstanding of the situation was evident.

(iv) A correct position vector for the speedboat was given by many candidates, although some omitted *t* from their expression.

(v) Those candidates who had given correct answers to parts **(iii)** and **(iv)** were often able to equate the vectors and solve for *t*, and these candidates usually went on to successfully find the position vector of the point of interception. Some candidates with incorrect answers to the previous parts attempted to solve their equations, although these sometimes resulted in negative values for time, or, if they equated both the **i** and **j** components, two different values for the time. Such candidates often did not go on to attempt to find the position vector of the point of interception. It must be pointed out that there were many candidates who, having found a correct time, did not go on and find the position vector of the point of interception, a case of not ensuring that all the demands of the question have been answered.

Solution: **(ii)** $40\mathbf{i}+96\mathbf{j}$ **(iii)** $(40\mathbf{i}+96\mathbf{j})+t(10\mathbf{i}+24\mathbf{j})$ **(iv)** $(120\mathbf{i}+81\mathbf{j})+t(-22\mathbf{i}+30\mathbf{j})$
(v) 18:30, $(65\mathbf{i}+156\mathbf{j})$

Question 11

In both parts **(a)** and **(b)** it was often difficult to be certain which values were being offered as solutions and which were part of the calculation process, especially in part **(a)** where $\tan x = 0$ gave $x = 0$ as one of the solutions.

(a) Full marks were obtained by only a few candidates. Most candidates were able to find at least one solution to the given equation. The main problems were as follows. Firstly, a term of $\tan x$ was sometimes cancelled out and the expression not factorised. Candidates should recognise that a quadratic should have two possible solutions. Secondly, one or two of the solutions were dismissed for unexpected reasons e.g. $\tan x$ cannot be negative, $\tan x = 0$ has no solutions. Finally, just considering $\tan x = -5$ and ignoring the fact that there is a solution in the second quadrant.

(b) Making use of the correct trigonometric identity as the first step was generally correctly done. Most candidates were able to solve the resulting quadratic equation correctly. However, many candidates still have problems dealing with negative values of trigonometric ratios often rejecting the result as being outside the required range. In this case, the equation $\sin y = -1$ was dismissed without consideration.

(c) Not all candidates recognised, or were able to deal with, the trigonometric ratio secant and were thus unable to make progress. Provided candidates considered $\cos^{-1} 0.5$ and equated it to $2z - \frac{\pi}{6}$, one correct solution was usually obtained. A pleasing number of fully correct solutions were obtained by many candidates, showing a good use of technique and understanding of radians. Answers in terms of π or in correct decimal form (3 significant figures) were equally acceptable.

Solution: **(a)** $0°, 180°, 101.3°$ **(b)** $30°, 150°, 270°$ **(c)** $\frac{\pi}{4}, \frac{11\pi}{12}$

ADDITIONAL MATHEMATICS

Paper 4037/21

Paper 21

Key Messages

Where an answer is given and a proof is required candidates need to be aware of the need to fully explain their reasoning and not jump to the answer. Where a solution would benefit from a diagram, candidates should be encouraged to concentrate on producing as clear a diagram as possible whilst avoiding using a scale diagram when instructed not to do so in the question. Candidates should take care in the accuracy of their answers and Centres would be advised to draw attention to the rubric which clearly states the requirements for this paper.

General Comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well presented, clearly organised answers. Presentation of answers was generally very clear to follow. Overall there was a full range of abilities in evidence. Questions which required the knowledge of standard methods were done well. The majority of candidates, as always, seem to try hard, but at times some produced mathematics that had little connection to the question being attempted or showed a misinterpretation of the information provided. These candidates need to improve their reading of questions and keep their working relevant. Candidates should always try to take note of the form of the answer required and where a specific method is indicated be aware that little or no credit may be given for alternatives. Candidates should also be aware of the need to use the appropriate form of angle measure within a question and to avoid mixing these as this can lead to further work being invalid.

When a good diagram would aid solution, candidates should be encouraged to produce one and when a sketch has been has been asked for in an earlier part, candidates should consider the relevance of this in subsequent work. Where a solution required several steps, clearly structured solutions were more likely to gain full credit, or partial credit when errors were made, than those which had apparently random expressions in no particular order or position on the page. Where a standard formula was to be applied, candidates were more likely to gain partial credit, when incorrect, if the formula was quoted first.

While candidates generally seemed able to apply the Quotient Rule for differentiation well in **Question 10(ii)** there was a much poorer understanding of the Product Rule on **Question 10(i).** The lack of a reasonable sketch on **Question 3(i)** often led to a loss of marks and to inappropriate work on **Question 3(ii)**. Similarly a diagram in **Question 9** would have helped on all three parts. There were parts of questions that candidates found more challenging – these being **Question 3(ii), Question 7(iii), Question 9(iii), Question 11(b)** and **Question 12(iv)**. The reasons for this were often misunderstanding mathematical rules or an omitting a statement of clarification.

Comments on Specific Questions

Question 1

Some candidates earned full marks for this short starting question. Other candidates gave the incorrect solution $x < 0$, $x < -1$ and some divided through by x at the start, immediately losing part of their solution and giving $x < -1$ only. Others, having removed the bracket and collected the terms, then treated those terms as a three term quadratic. Where candidates found both parts of the inequality correctly, a single combined inequality was common.

Answer: $-1 < x < 0$

Question 2

This standard type of question was very well done and there were many fully correct answers. There were few calculation errors and, apart from a few candidates who tried to deal with the negative index by multiplying out, an attempt at rationalisation was usually made. The most frequent error was in moving the 6 to the denominator when dealing with the negative index.

This question clearly stated that calculators should not be used and it is therefore important that candidates show sufficient working when multiplying out brackets. Jumping to the answer without this resulted in a loss of marks.

Answer: $6 - 3\sqrt{3}$

Question 3

(i) While there seemed to be reasonable understanding of what a modulus graph should look like, there were a significant number of candidates who produced a standard quadratic graph. Commonly, candidates attempted to place the maximum at the *y*-intercept and often distorted the natural shape to try to achieve this. Candidates also, on occasion, omitted one end of the graph and occasionally the roots were shown as 2 and 4.

(ii) Very few candidates made use of their sketch from part **(i)** which would have led to a quick and exact solution by inspection. The key point was the maximum of the modulus graph which could be identified by symmetry avoiding any complicated algebra. Much, usually irrelevant, work of no credit was often presented. The most common misunderstanding was to attempt to solve the modulus of the quadratic. A few candidates who identified $k = \pm 9$ managed to arrive at the correct inequality. Others either disregarded the zero limit or included the equality on one or both sides.

Answer: **(ii)** $0 < k < 9$

Question 4

This was a successful question for most candidates. It was a standard question requiring application of the Factor Theorem and the Remainder Theorem and, as such, most applied these appropriately and solved the resulting quadratic equations correctly. The most common errors were arithmetical. Division was seen, but rarely led to a full and final solution. Indeed, this method usually resulted in not even partial credit being awarded as the division needed to be completed using *a* and *b* and equating to zero or –12 to progress.

Answer: $a = -3$, $b = -23$

Question 5

(i) The methods of completing the square and equating coefficients were equally popular amongst good solutions. When completing the square, candidates sometimes needed to be more careful with brackets and with dealing with the multiplication of the constant term by 2 at the appropriate stage. There were still many answers given in the wrong format despite the required format being clearly stated.

(ii) This part specifically stated 'hence' and a number of candidates ignored their answer to part **(i)** and used calculus to start again. This could be treated as checking their earlier work if correct but was penalised if it did not match part **(i)**. Other common errors were to switch the minimum value and the corresponding *x*-coordinate.

Answers: **(i)** $2\left(x - \frac{1}{4}\right)^2 + \frac{47}{8}$ **(ii)** $\frac{47}{8}$ when $x = \frac{1}{4}$

Question 6

(a) The majority of candidates answered this question well. Some gave the complete expansion rather than picking out the required term, which wasted time. Most candidates gave an unsimplified version first and sensibly evaluated ${}^nC_r$ terms and terms involving indices separately before arriving at the final answer. The most common errors were in omitting the minus sign or not raising the 2 to the power 5.

(b) (i) This was well done by many. Some candidates gave the coefficient of the squared term as 30. Less frequently, candidates omitted the constant term or gave the expansion in descending powers. As in part **(a)** it was better for candidates to give the unsimplified version first as this could gain partial credit.

(ii) This was again well attempted by many, with candidates showing sufficient working to gain credit for using the values in their expansion. At times the x and x^2 were not removed to allow a sensible solution. Multiplying the wrong coefficient by 1.5 was relatively rare and nearly all candidates who arrived at the correct equation solved it correctly.

Answers: **(a)** –48384 **(b)(i)** $1 + 12x + 60x^2$ **(ii)** – 4

Question 7

(i), (ii) These were both almost universally well attempted. Candidates dealt well with the fractional and negative indices. There were relatively few errors and those made involved logs and increasing indices. Otherwise errors tended to involve numerical slips.

(iii) This part of the question proved to be more challenging for candidates. While the majority knew to equate part **(i)** to zero, a small minority equated part **(ii)** to zero. Many candidates need to improve their algebraic skills, with weaknesses in this area being highlighted by their attempts at solution. Some candidates were fortuitously arriving at a 'correct' x value of 1. Several candidates solved the equation by inspection at an early stage in their work, which was acceptable. Partial credit was given for attempting to find the coordinates of the stationary point and then finding the value of the second derivative using that point. Some, otherwise good, solutions were spoiled by not explicitly stating that their calculation gave a positive value and that this was the reason for it being a minimum.

Answer: **(i)** $-\frac{1}{x^2}+\frac{1}{x^{\frac{1}{2}}}$ **(ii)** $\frac{2}{x^3}-\frac{1}{2x^{\frac{3}{2}}}$ **(iii)** Minimum at $x = 1$, $y = 3$

Question 8

(i) The majority of candidates attempted this part very well with both arc length and area of a sector formulae applied correctly. Most candidates gave sufficient and clear working to justify arriving at the given answer. Some candidates attempted to manipulate incorrect expressions to the required result. The alternative approach of using $A = 0.5 \times r \times \theta$ was seen a number of times and was nearly always applied correctly. A few equated the arc length to 30 rather than the perimeter which was unfortunate as they often had both correct formulae within their solution.

(ii) Candidates who were unsuccessful in part **(i)** sometimes made no attempt to answer part **(ii)**. Candidates should be aware that, when a formula is given, it is usually possible to proceed with the next part of the question, even if the attempt to prove the formula in the previous part has not been successful. A significant number of candidates did not know to differentiate and a small number that differentiated did not know to equate it to zero. Other candidates stopped at $r = 7.5$ and as always, a careful reading of the question is recommended. It was possible to complete the square but this was rarely seen. It should also be noted that the answer was exact and so did not require to be given to 3 significant figures.

Answer: **(ii)** 56.25

Question 9

Generally this question divided candidates between those who followed the order suggested by the question parts and those who, usually on the basis of false assumptions on the geometry of a kite, sought to find p and q first. A reasonable sketch would have been beneficial but was not seen often enough and when seen frequently as an afterthought when attempting the area.

(i) A large number of candidates found the required coordinates quite straightforwardly by finding the midpoint of *BD* as was expected. There were some correct answers found by more circuitous routes. A common misconception was to assume *E* was the midpoint of *AC* and to try to find *p, q* and/or the equation of *AC* first. Whilst this was possible using perpendicular gradients it involved much work and usually proved unworthy of credit due to a conceptual error at some stage.

(ii) Many candidates stated that the gradient of *AC* was 2, sometimes based on work done in part **(i)**. A large number either used incorrect values for *E* or their values for *p* and *q* which should have been derived from their equation and were not needed until part **(iii)**. There were a few incorrect methods used to find gradients and perpendicular gradients but these formulae were generally well known.

(iii) There were a good number of well-presented and concise solutions to this part of the question. There were also many solutions which made little attempt to identify which side lengths or triangle areas their calculations referred too, making it very difficult to award marks for method when an incorrect area was ultimately found. The methods for correctly finding the area of a triangle were numerous and used in equal measure. The most common error was to omit division by 2. A small minority of candidates continued to work with the unknowns *p* and *q*.

Answers: **(i)** (3, 5) **(ii)** $y - 5 = 2(x - 3)$ **(iii)** *15*

Question 10

Generally candidates were more successful using the quotient rule rather than the product rule. This was, in part, due to better differentiation of the components. It was noticeable that candidates were also more likely to quote the quotient rule. In both parts candidates were more likely to be awarded full marks and certainly to gain partial credit where their method was clear; quoting the rule clearly and showing the component differentiations separately. It is difficult, otherwise, to determine whether candidates have made sign or coefficient errors or do not know the appropriate rule.

(i) There were many fully correct answers to this part. Most candidates knew, for example, that the derivative of sin *x* was cos *x*. Candidates were less certain in dealing with the 2*x* and $\frac{x}{3}$. The most common error was in multiplying the two derived components together with no evidence of the product rule whatsoever.

(ii) The component parts were nearly always differentiated correctly leading to a large number of fully correct answers. Occasionally $\text{cosec}^2 x$ was given rather than $\sec^2 x$ and 1 was sometimes differentiated to 1. The product rule was seen on quite a number of scripts and was usually applied correctly. Poor bracketing was an occasional issue although often this was clarified by a subsequent multiplication.

Answers: **(i)** $\frac{1}{3}\cos 2x \cos\left(\frac{x}{3}\right) - 2\sin 2x \sin\left(\frac{x}{3}\right)$ **(ii)** $\frac{(\sec^2 x)(1+\ln x) - \frac{1}{x}(\tan x)}{(1+\ln x)^2}$

Question 11

(a) This was one of the best attempted questions on the paper with most candidates scoring most if not all of the marks even if they found it difficult to score elsewhere. Many solutions were concise, with candidates realising that 64 was a power of 2 and quickly forming a quadratic equation which in turn was usually factorised correctly. Some candidates assumed that an index equation required logs, which is often the case, although here this usually led to more work, with mixed results.

(b) There were some very well presented and concise solutions by those candidates who not only knew the change of base rule but also the other key laws of logarithms. The solution generally hinged on realising that $\log 2a = \log 2 + \log a$ and in working in base *a* as quickly as possible. Many candidates applied their own incorrect variations on the laws leading to many poor solutions occasionally gaining partial credit by changing base correctly at the outset.

Answers: **(a)** $x = 2$ or $x = 3$ **(b)** $\log_a 4$

Question 12

(i) This part was very well done by the majority of candidates with most gaining full marks. Calculating g(8) first and finding fg(x) first were equally common and the omission of +1 in the denominator the only common error.

(ii) This part was also well attempted. Inevitably, there were many candidates who multiplied f(x) by itself. The majority who appreciated that composition was required usually made the initial substitution correctly. Dealing with the denominator proved the biggest challenge and on occasions having simplified both numerator and denominator correctly, the logical step of cancelling the $x + 1$ was either omitted or only carried out by a lengthy process involving multiplying out then factorising first.

(iii) Most candidates arrived at the inverse function successfully. Stating the domain and range were less well done with even more able candidates finding it difficult to find appropriate values or using poor notation in terms of x and $g^{-1}(x)$. The connection between the given domain for g(x) and the range of the inverse was seldom recognised.

(iv) There were few fully correct answers to this part seen. Many candidates found it difficult to score at all and a very few did not attempt an answer. Graphs were often poor with straight lines often presented. Even better candidates drawing curves often omitted the sections of the graphs in the second and fourth quadrants. Another frequent error was to extend the curves beyond -1 on each axis and into the third quadrant.

The geometric relationship required, that the function and the inverse were reflections in the line $y = x$, was frequently implied. In this question an explicit statement was needed. Candidates who drew the line $y = x$ may have been credited if it had been labelled. Equally, candidates should be aware that, in cases when mirror images meet, the point of intersection should be on the mirror line.

Answer: **(i)** $\frac{6}{4}$ **(ii)** $\frac{4x}{3x+1}$ **(iii)** $g^{-1}(x) = x^2 - 1$; Domain $x > 0$; Range $g^{-1}(x) > -1$

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key Messages

In order to do well in this examination, candidates needed to give clear and well thought out answers to questions, with sufficient method being shown so that method marks can be awarded.

General Comments

A good number of candidates gave clearly presented answers. Other candidates need to appreciate that poorly presented work is often difficult to credit. In order to improve, some candidates need to understand that their working must be detailed enough to show their method clearly. This is even more important if they make an error. Showing clear method is very important if a question starts with the words "Show that…". This indicates that the answer has been given to the candidates and that the marks will be awarded for showing how that answer has been found. Showing clear method is also very important if the use of a calculator is prohibited. The need for this was highlighted in **Questions 1** and **3** in this examination. Occasionally it was evident that candidates needed to read the question more carefully. This was exemplified in **Question 5** where candidates often misread the figures and **Question 8** where an exact answer was required. Candidates may benefit by taking more care with their arithmetic and avoiding the premature approximation of answers. Candidates may also improve their performance if they take notice of key instructions in questions – such as "Hence..." in **Question 4(ii)** and "Use the graph..." in **Question 10(ii)**.

Comments on Specific Questions

Question 1

This question proved to be an accessible start to the paper for almost all candidates. Most candidates gave clear evidence of method, as was required since the use of a calculator was not allowed in this question. A very few candidates used the method of simultaneous equations with the vast majority opting to square and rationalise in the expected way. A very few candidates attempted to rationalise using $\frac{\sqrt{5}+1}{\sqrt{5}-1}$ or $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ and a very few omitted the cross terms when squaring $2+\sqrt{5}$. Generally, this was a good start to the paper.

Answer: $\frac{29}{4}+\frac{13}{4}\sqrt{5}$

Question 2

This question was generally well answered by a good many candidates.

Most candidates chose to eliminate y and did so correctly by substitution. Most of these were able to collect like terms successfully. A fair number of candidates had a sign error in the x term and scored method marks if they then considered the discriminant and attempted to solve their resulting quadratic. Candidates should give attention to the presentation of their answers as poor presentation sometimes resulted in candidates miscopying their own writing and making errors. Candidates who attempted to eliminate x rarely made any real progress.

Candidates who chose the alternative calculus method commonly made sign errors. On the whole this method was significantly less successful. It was common to see an error in the first step with $k = 4x - 9$ being stated rather than $-k = 4x - 9$. Whilst this was often substituted into $y = -kx + 2$ it was not always then equated to $2x^2 - 9x + 4$. A small number of candidates used the approach of constructing the equation of a tangent to the curve through a general point with, for example, coordinates $\left(m,\ 2m^2 - 9m + 4\right)$ and equated the constants term of this with 2 from the given form. This, slightly more complicated, approach was successful as long as candidates presented their steps logically.

Answer: 5 and 13

Question 3

(i) Candidates almost always showed sufficient method to earn the mark.

(ii) Again candidates did well here – many used long division, synthetic division or equated coefficients and did so correctly. A few candidates made sign errors or made no real progress with this part. A small number of candidates found that $(x - 5)$ was a factor and found the quadratic factor that paired with that – thereby not answering the question being asked.

(iii) Occasionally candidates did not show method and were penalised a mark. The question required the use of their answer to part **(ii)** and so candidates needed to show evidence of how this was done to score. Otherwise this part was well answered.

Answer: **(ii)** $3x^2 - 17x + 10$ **(iii)** $-1, 5, \frac{2}{3}$

Question 4

(i) This was reasonably well done. Candidates who could have improved, generally should have been more careful in finding the value of *r*, as this was often incorrect. Some candidates started using a common factor of 2, 3 or 6 rather than 12. Some candidates gave their solutions in an incorrect format, even though the required structure was given in the question.

(ii) Candidates need to appreciate that when part of a question starts with the word "hence" it is expected that the previous part of the question be used in answering the part under consideration. Many candidates restarted this part using other methods and were only credited if their values agreed with the values they had found in part **(i)**. It was unfortunate that some arrived at the correct values in part **(ii)** having made an error with the value of *r* in part **(i)**, but then omitted to return to part **(i)** and correct the error they had made there.

A small number of candidates thought the values of *x* and "*y*" should be swopped because it was an "inverse" or reciprocated the value of *x* as well as the least value.

Answer: **(i)** $12\left(x - \frac{1}{4}\right)^2 + \frac{17}{4}$ **(ii)** $\frac{4}{17}$ when $x = \frac{1}{4}$

Question 5

(i) Most candidates answered this well. There were occasional sign errors and a very few candidates did not progress beyond the first, completely unsimplified, line.

(ii) A high proportion of candidates found the value of *a* correctly with only occasional sign errors being seen. Some candidates misread the –23 as 23. The –20*a* element or the 160 element of the coefficient of *b* were sometimes omitted. A small number of candidates misunderstood the correct method and attempted to work with $\left(1 - 23x + 222x^2\right) - \left(1 - 20x + 160x^2\right)$.

Answer: **(i)** $1 - 20x + 160x^2$ **(ii)** $a = -3$, $b = 2$

Question 6

(a) (i) This was well answered, although occasional answers of $u = 0$ or 10 were given.

(ii) Many candidates gained one mark only from considering only one of $2x + 3 = 1$ or $2x + 3 = -1$. Others thought the modulus should be part of the answer and, after $x = -1$ had been found, gave the answer $x = 1$. Some candidates discarded one of the answers, usually, $x = -2$ as this resulted in a negative value that candidates decided was invalid, ignoring the modulus. A small number of candidates squared both sides and solved the resulting quadratic. Some candidates used $2x + 3 = 0$ or made some spurious use of logs in their solution and made no progress.

(b) A good number of candidates gave clear, concise and fully correct answers using the expected change of base and laws of logarithms. However, some answers were obtained fortuitously from 'double errors' made by misusing log laws. In order to improve, candidates must have a clear understanding of the origins and use of laws of logarithms so that this type of error is avoided. Some candidates did not identify $\log_a 5$ as the sensible term to start with and instead changed $\log_3 a$ to $\dfrac{1}{\log_a 3}$. After this, some were successful, but others misapplied log laws and produced something along the lines of $\log_a 5 - 3$. A small, but not insignificant, number of candidates also lost the last 2 marks through invalid log moves such as $\log_3 225 - (\log_3 5 - \log_3 a + \log_3 a)$. Most candidates were able to score 1 or 2 marks, either for log 225 or for changing the base by some valid method or both.

Answer: **(a)(i)** 1 **(ii)** –1, –2 **(b)** $\log_3 45$

Question 7

(i) Most candidates scored full marks here. A small number differentiated each part of the product and did not apply the product rule, but on the whole the question was well answered.

(ii) Again, most candidates scored full marks here. A small number earned one mark only for a reasonable attempt at the chain rule. Some candidates produced answers which involved $\dfrac{1}{\ln(\cos x + 2)}$ or $\dfrac{1}{(\cos x + 2)^2}$ or $\dfrac{1}{-\sin x}$.

(iii) A good number of candidates scored all 3 marks here – and generally the quotient rule was the method used. A few did attempt the chain and product rules, but were less successful as the use of the chain rule introduced another level of difficulty. Some candidates omitted the brackets around the $1 + \sqrt{x}$ in the quotient rule and never implied them in any further working. Candidates should be aware that this type of omission results in their answer being incorrect and they are therefore penalised. A small number of candidates misremembered the quotient rule with the terms in the numerator being reversed or the terms being summed rather than a difference being used.

Answer: **(i)** $x^4(3e^{3x}) + 4x^3e^{3x}$ **(ii)** $\dfrac{1}{2 + \cos x} \times (-\sin x)$ **(iii)** $\dfrac{(1+\sqrt{x})\cos x - (\frac{1}{2}x^{-\frac{1}{2}})\sin x}{(1+\sqrt{x})^2}$

Question 8

A very well answered question by the large majority of candidates. Occasionally errors were made in expanding the brackets after the correct substitution had been made, omitting the cross terms. A few candidates made sign errors. A few candidates forgot to find the length of *AB*, whilst others did not pay heed to the word **exact** in the question and gave their answer as a decimal. Those few candidates who did not score full marks could improve by being more careful with their method and reading the question more carefully.

Answer: $\sqrt{72}$

Question 9

(i) A good number of candidates appreciated the need to integrate and did so correctly. Some of these omitted the constant of integration, however, or omitted to divide by 2 and therefore lost marks. Some candidates calculated the value of $\frac{dy}{dx}$ when $x = 4$ and then used $y = mx + c$.

(ii) A good number of correct answers were given once again. Many appreciated the need to integrate the equation they had found in part **(i)**. Many other candidates did not see the need to integrate their y and often integrated $\frac{dy}{dx}$ or did not integrate their answer to part **(i)** and chose to substitute the limits directly into it instead. Occasionally, those candidates who used $\frac{1}{3}$ as a multiplier to simplify their integration, generally a useful tactic, made an error in dealing with the constant – for example $\frac{1}{3}\left[\frac{(2x+1)^{\frac{5}{2}}}{5}+x\right]$ was seen on more than one occasion.

Answer: **(i)** $y = \frac{(2x+1)^{\frac{3}{2}}}{3} + 1$ **(ii)** $\frac{107}{30}$

Question 10

(i) Most candidates realised the need to take logarithms. A few candidates treated the equation as if it were $\log y = \log(Ab)^x$ and then gave subsequent steps $\log y = x\log(Ab) = x\log A + \log b$ which had unfortunate consequences in part **(ii)**. It was clear that some had memorized the correct form and simply quoted it. Some candidates took logs to base b which, though not incorrect, was not particularly helpful in part **(ii)**.

(ii) Many fully correct answers were seen, using the expected methods as indicated in the question – that is using the gradient and intercept of the given graph. Some candidates misread the scale and the intercept being given as 0.6 instead of –0.6 was fairly common. Most candidates understood the need to find the gradient and intercept of the line. Some candidates then quoted these as the values of A and b rather than anti-logging them. Other candidates used logarithms to base 10 rather than base e, which was not valid in this part. Those choosing to attempt simultaneous equations often lost accuracy or used values that were invalid for the model. This was the least successful method chosen. Many candidates quoted $A = 1$ without indicating that $A = e^{-0.6}$ or a more accurate decimal and therefore lost marks. In order to improve, candidates should realise that, when they are requested to round an answer, it is sensible to quote the value to a reasonable accuracy before it is rounded to ensure that method marks at least can be awarded.

(iii) The quickest and easiest method of solution – that is using the graph – was, most often, overlooked. Many candidates earned the method mark for using their values from part **(ii)**. Some confused the models and were using 220 rather than ln220 in the log form of the model or 5.39... in the exponential form.

Answer: **(i)** $\log y = \log A + x\log b$ **(ii)** $A = 0.5$, $b = 4$ **(iii)** 4.4

Question 11

(i) A good number appreciated the importance of the domain and used it correctly to find the answer. A fair few candidates gave the answer $f(x) \geq 7$ following the misconception that $x > 2$ meant that $x \geq 3$. Some candidates ignored the given, restricted domain and gave the answer $f(x) > -1$. A small number of candidates confused the domain and range and gave an inequality in x.

A good number of candidates made a correct start to the method of finding the inverse function. A good number of these did find the inverse function successfully. Some candidates omitted the brackets around the x + 1 and therefore lost a mark. Some candidates "cancelled" the logs in their solution and gave an answer of $f^{-1}(x) = \frac{\cancel{\log}(x+1)}{\cancel{\log}\ 2} = \frac{x+1}{2}$.

(ii) A good number of candidates also appreciated the connection between the domain and range of a function and its inverse. Some candidates did not attempt to state the domain and range and other candidates thought that the domain was $x > 2$ and calculated the range using that, $f^{1}(x) > 1.58$ was not an uncommon answer for the range. Some candidates confused the notation required for the domain and range and this was penalised.

(iii) This question proved to be a good discriminator. A high proportion of candidates earned the first mark by forming the correct composite function – although a few attempts at fg(x) were seen. A good number went on to earn full marks considering both factors of the composite function and explaining correctly why one factor led to no solution and the other resulted in a solution outside the domain. Some candidates multiplied their factors together and consider one term only. These lost a mark as their explanation was not complete. Some candidates thought that it was not possible for 2x to be equal to x or divided through by x and commented that 2 ≠ 1 and so no solution, or omitted to simplify $2x - x = 0$ to $x = 0$. Generally these candidates were not realising that they were undertaking the equivalent of dividing by 0. Division rather than factorisation should be discouraged as solutions are often lost in this way.

Answer: **(i)** $f(x) > 3$ **(ii)** $f^{-1}(x) = \log_2(x+1)$, $x > 3$, $f^{1}(x) > 2$ **(iii)** $gf(x) = 2^x(2^x - 1)$

Question 12

(i) Candidates found this question straightforward and the first three marks were usually earned without difficulty. Correct answers were often spoilt by attempting to combine into a single, spurious or incorrect inequality. Many candidates did not draw a sketch, which would have proved helpful, and either stated $x = 2$, $x = 4$ or gave an incorrect inequality. It is unclear as to whether this was because candidates had misinterpreted the meaning of the inequalities or if they had misinterpreted the question.

(ii) Most candidates substituted into the correct $\frac{dy}{dx}$, although a few used $x^2 - 6x + 8$. Some used the x-coordinates 2 and 4 they had found in part **(i)** and then calculated the y-coordinates and the gradient of the chord connecting these points. Finding the gradient of the normal was generally well done. The majority of candidates found the correct y-coordinate for $x = 3$, although a few miscalculated the value as 0. The method of finding the equation of the line was obviously well understood. Occasionally careless arithmetic errors in finding the value of c, when using $y = mx + c$, spoilt some otherwise good answers. Some candidates forgot to state the coordinates of P. Other candidates found the x-intercept rather than the y-intercept. These candidates could, perhaps, have been helped by reading the question a little more carefully or rereading the question once it had been completed.

Answer: **(i)** $x \le 2$, $x \ge 4$ **(ii)** $y = \frac{1}{3}x + 17$, $P(0, 17)$